Quiz 4

#1 ∇f should be perpendicular to level sets, eliminating first two answer choices.

$|\nabla f|$ is the rate of increase (in the direction of steepest increase) and so the third answer choice is a lot more reasonable than the fourth.

#2 Told that $f_x(\pi/2, 0) = 0$

$$f_y(\pi/2, 0) = 0.$$

$$g(x,y) = \sin(f(x,y))$$

$$g_x(x,y) = \cos(f(x,y))\, f_x(x,y)$$

$$g_y(x,y) = \cos(f(x,y))\, f_y(x,y).$$

What can I plug in for x and y to make these zero?

A: Just use $(\pi/2, 0)$.

Suppose $(2, 4)$ is a critical point of $f(x, y)$. Which of the following must be a critical point of $g(u, v) = \sin(f(u + v, u - v))$?

$(6, -2)$	**A**
$(3, -1)$	**B**
$(\sin(2), \sin(4))$	**C**
$(\sin(6), \sin(-2))$	**D**
None of the above	**E**

Total Results: 11

We're told:

$$f_x(2,4) = 0 \qquad f_y(2,4) = 0.$$

$$\begin{tikzcd}[column sep=small]
 & & & g(u,v) \arrow[dll, dash] \arrow[drr, dash] & & & \\
 & x \arrow[dl, dash] \arrow[dr, dash] & & & & y \arrow[dl, dash] \arrow[dr, dash] & \\
u & & v & & u & & v
\end{tikzcd}$$

$x = u+v$

$y = u-v$

$$g_u(u,v) = \cos\left(f(u+v,u-v)\right) f_x(u+v,u-v)\cdot(1)$$

$$+ \cos\left(f(u+v,u-v)\right) f_y(u+v,u-v)\cdot(1)$$

$$g_v(u,v) = \cos\left(f(u+v,u-v)\right) f_x(u+v,u-v)\cdot(1)$$

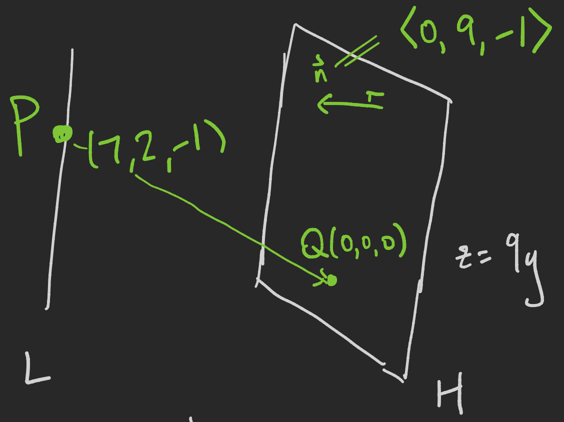
$$+ \cos\left(f(u+v,u-v)\right) f_y(u+v,u-v)\cdot(-1)$$

What can we plug in for u and v to make these zero?

Want to make $u+v=2$

$u-v=4$

$\rightsquigarrow$ $u=3 \qquad v=-1.$



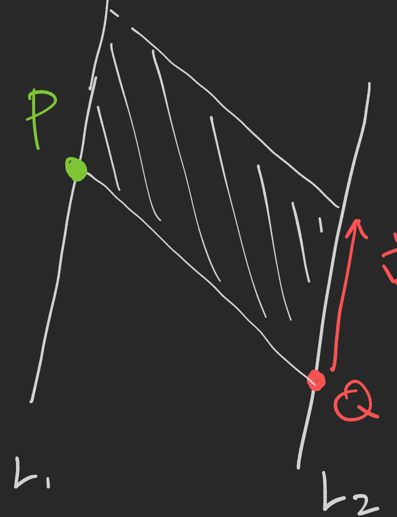
$$\frac{x-7}{3} = y-2 = \frac{z+1}{9}$$

If L and H are parallel, then distance between them is the distance between P and H, where P is any pt. on L.

For distance from P to H, can use formula, or

pick some point Q on H e.g. $(0,0,0)$ in this example, then compute

$$\left| \operatorname{comp}_{\vec{n}} \overrightarrow{PQ} \right|.$$



If L_1, L_2 are parallel lines, to find distance, pick a point P on L_1. Then it's just the distance from P to L_2.

Q is any pt. on L_2, $\vec{v}$ is a direction vector for L_2.

$$\text{Distance} = \frac{|\vec{v} \times \overrightarrow{PQ}|}{|\vec{v}|}$$

← Area of shaded parallelogram

⚠ If dealing with skew lines, refer to last example in §12.4

Consider the polar curve $r = \sin(3\theta)$. Starting from $\theta = 0$, when does this polar curve begin to retrace itself?

At $\theta = \pi/3$

At $\theta = 2\pi/3$

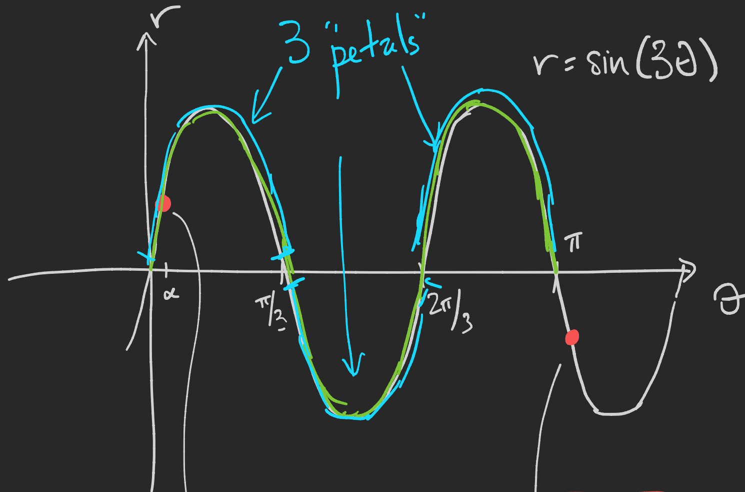
At $\theta = \pi$

At $\theta = 2\pi$

This curve does not retrace itself.

Consider the polar curve $r = \sin(4\theta)$. Starting from $\theta = 0$, when does this polar curve begin to retrace itself?

- At $\theta = \pi/4$
- At $\theta = \pi/2$
- At $\theta = \pi$
- At $\theta = 2\pi$
- This curve does not retrace itself.



$\theta = \alpha$
$r = \sin(3\alpha)$

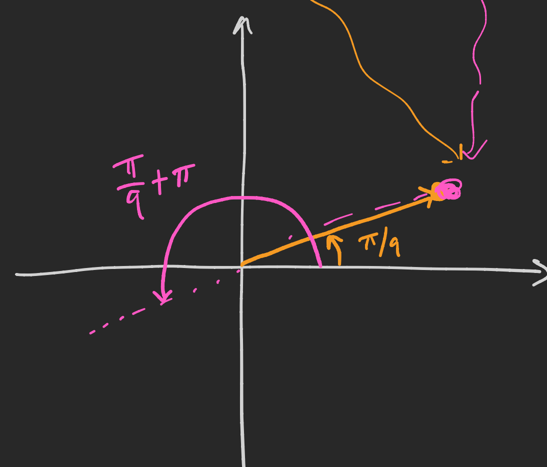
$\theta = \alpha + \pi$
$r = \sin(3(\alpha+\pi))$
$= \sin(3\alpha + 2\pi + \pi)$
$= -\sin(3\alpha)$

same point in the xy-plane !!!

example:

$\theta = \frac{\pi}{9}$
$r = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{9} + \pi$
$r = -\frac{\sqrt{3}}{2}$



For $\sin(4\theta)$, turns out you need 2π rather than π.